Recent Lattice QCD results and implications for BES

Sayantan Sharma



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Bielefeld-BNL-CCNU collaboration

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Outline

1 The QCD phase diagram: outstanding issues from lattice

2 Equation of state at finite μ_B

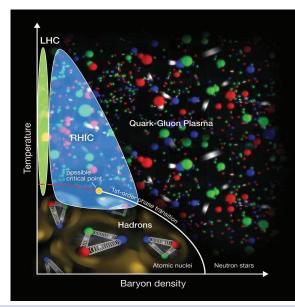
Freezeout and Lattice observables

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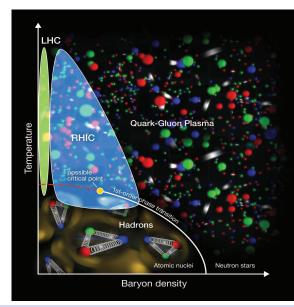
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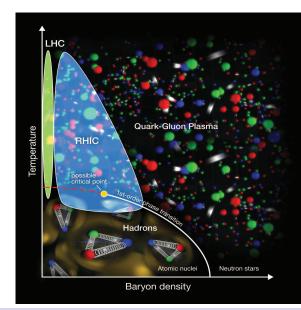
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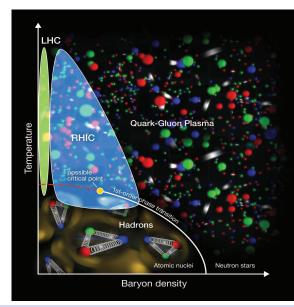
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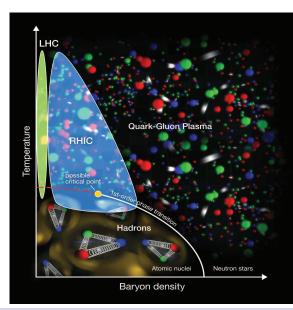
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- Understand what happens to HRG picture at finite μ_B.
- Bracket the position of CEP in phase diagram.
- Understand the critical behavior due to the light quarks in the crossover region.



• One of the methods to circumvent sign problem at finite μ : Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T .

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0, T)}{T^4} + \frac{1}{2} \left(\frac{\mu_B}{T}\right)^2 \chi_2^B(0, T) + \frac{1}{4!} \left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \dots$$

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 χ_8^B for $N_{ au}=8$ pure staggered fermions[Gavai& Gupta, 08]. χ_6^B for $N_{ au}=6,8,12,16$ HISQ fermions

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• These observables imp. for EoS $\to \chi_6^B$ can already constrain QCD pressure in the regime approximated by Hadron Resonance gas model.

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 - Matrix inversions increasing with the order
 - Delicate cancellation between a large number of terms for higher order QNS.

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- Calculating explicitly the lowest eigenvalues improves performance of the fermion inverter. Optimized codes developed to this end.
- Efficient codes based on modern computer architectures are being developed. [O. Kaczmarek, C. Schmidt, P. Steinbrecher, M. Wagner, 14]

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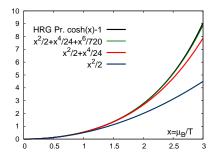
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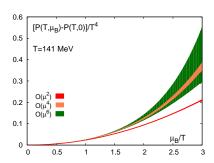
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Constraining EoS

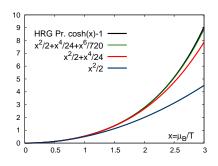
• In a regime where Hadron Resonance gas is anticipated to be a good description of QCD, including χ_6^B term already reproduces $P(\mu_B)$ within 5% accuracy.

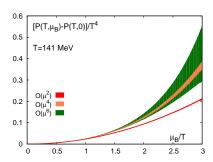




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- In a regime where Hadron Resonance gas is anticipated to be a good description of QCD, including χ_6^B term already reproduces $P(\mu_B)$ within 5% accuracy.
- We are improving the errors on $\chi_6^B \to$ increase statistics twofold this year.



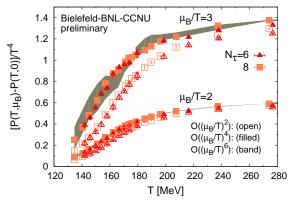


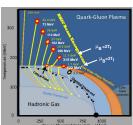
EoS away from criticality

• The pressure for T>160 MeV already constrained by χ_B^6 for $\mu_B/T\leq 2 \to \text{input}$ for hydrodynamic modeling of QGP.

EoS away from criticality

- The pressure for T > 160 MeV already constrained by χ_B^0 for $\mu_B/T \leq 2 \rightarrow$ input for hydrodynamic modeling of QGP.
- Extension to $\mu_B/T \sim 3$ is in progress.





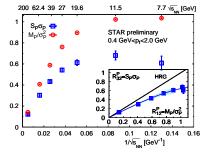
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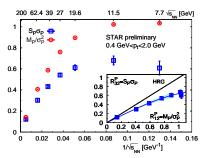
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$$\frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2} \left(\frac{\mu_B}{T}\right)^2 \left[1 + \frac{1}{12} \frac{\chi_2^B}{\chi_4^B} \left(\frac{\mu_B}{T}\right)^2 + \dots \right] .$$



Clear deviation from HRG predictions!

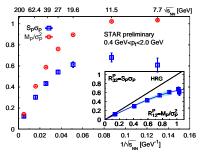
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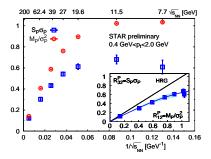
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$$S_B \sigma_B = \frac{\mu_B}{T} \frac{\chi_B^B}{\chi_B^B} + \mathcal{O}\left(\frac{\mu_B}{T}\right)^3$$

= $\frac{M_B}{\sigma_B^2} \frac{\chi_B^B}{\chi_B^B} + \dots$

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• Strangeness neutrality condition: $\frac{n_p}{n_p+n_p} = 0.4$.

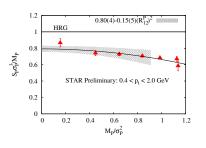
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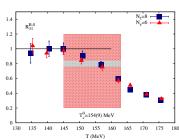
Is this deviation consistent with the trend from Lattice?

$$\frac{S_B \sigma_B^3}{M_B} = \frac{\chi_2^B}{\chi_4^B} + \frac{1}{6} \left[\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_2^B}{\chi_4^B} \right)^2 \right] \left(\frac{M_B}{\sigma_B^2} \right)^2.$$

$$R_{31}^B = R_{31}^{B,0} + R_{31}^{B,2} \left(\frac{M_B}{\sigma_B^2} \right)^2. \text{ [Karsch et. al., arxiv:1512.06987]}$$

• Experimental data consistent with QCD prediction. Caveat $n_P \neq n_B!$



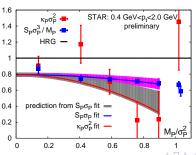


More Observables?

NLO
$$\kappa_B \sigma_B^2$$
 for $\mu_Q \sim \mu_S \sim 0$: $R_{42}^{B,2} = \frac{1}{2} \left| \frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_2^B}{\chi_4^B} \right)^2 \right| = 3R_{31}^{B,2}$.

- Fit to experimental data shows these quantities are closely related.
- $R_{31}^{B,0} \approx R_{42}^{B,0}$. At NLO consistent within large errors in the data. [

Bielefeld-BNL-CCNU collaboration, In preparation, 16]

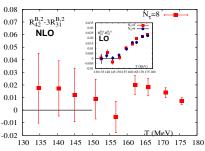


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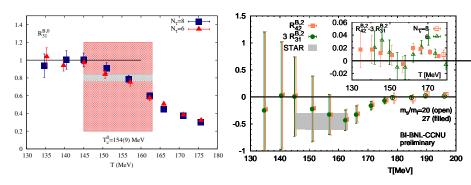
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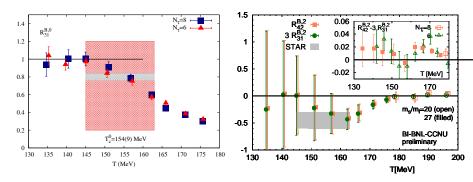
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- Aiming for a factor two reduction of errors on 6th order cumulants near T_c .



• Freezeout curve parametrized as $T = T_{f,0}(1 - \kappa_2^f \mu_B^2/T_{f,0}^2)$.

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- Expanding the observable about the freezeout surface at $\mu_B=0$, $\Sigma_r^{QB}(\mu_B)=\Sigma_r^{QB}(0)+\left[\Sigma_r^{QB,2}-\kappa_{\mathbf{2}}^f\ T_{f,0}\frac{d\Sigma_r^{QB,0}}{dT}|_{T_{f,0}}\right]\frac{\mu_B^2}{T^2}\;.$

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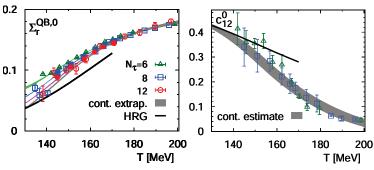
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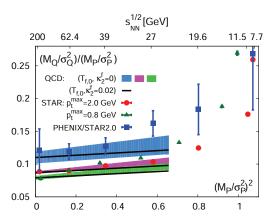
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 ight)^4$.
- An estimate of Σ_r^{QB} and R_{12}^B from experiments allows us to calculate C_{12} . [Bielefeld-BNL-CCNU collaboration, 15]

- Caveat: In experiments one measures protons Σ_r^{Qp} , R_{12}^p . Need to understand proton vs baryon number distributions. [Asakawa & Kitazawa, 12]. Within HRG at least R_{12}^B is mimicked by R_{12}^P within 10%.
- Additionally take into account also corrections due to finite range of momenta of detected particles.

[Karsch, Morita and Redlich, 15, P Garg et. al., 13, Bzdak & Koch, 12].

• From the 2 independent expressions of Σ_r^{QB} we extract $c_{12}(T_{f,0}, \kappa_2^f) = c_{12}(T_{f,0}) - \kappa_2^f D_{12}$.





• This excercise give $T_{f,0} = 147$ MeV consistent with expectation that its at or below T_c .

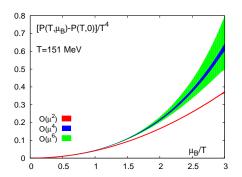
Curvature: $\kappa_2^f < -0.012(15) \rightarrow$ near to chiral curvature $\kappa_2^B = 0.0066(7)$.

[Bielefeld-BNL-CCNU collaboration, 15]

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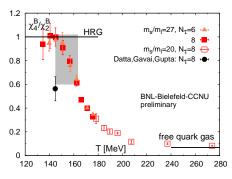
Breakdown of HRG

- Breakdown of HRG+ onset of criticality can be already constrained with χ_6^B .
- ullet Near critical point all terms in the Taylor expansion nearly equal o need to improve the errors to observe!
- At CEP: $\chi_n > 0$, $\kappa_B \sigma_B^2 > 1$



Critical-end point search from Lattice

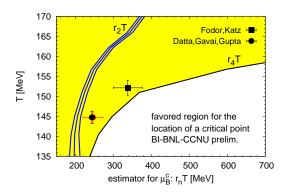
- Radius of convergence: $r_{2n} \equiv \sqrt{2n(2n-1)\left|\frac{\chi_{2n}^B}{\chi_{2n+2}^B}\right|}$.
- Only existing result $T_{CEP} = 0.94 T_c$ $\mu/T = 1.68(5)$ [S.Datta, R. Gavai, S.Gupta, 13, Mumbai group]
- Lowest r_{n=2} varies significantly from our estimates and HRG → lattice cut-off effects needs to be considered!

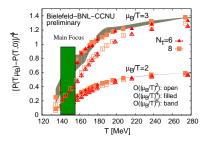


Critical-end point search from Lattice

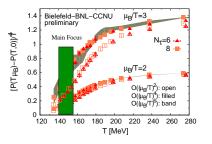
- Current errors on our χ_6^B/χ_4^B only allow us to define a favored region for CEP!
- χ_8^B measured to get errors bounds on radius of convergence estimates.
- Connection to experiments non-trivial due to non-equilibrium effects.

[S. Mukherjee, Y. Yin, R. Venugopalan, 15]

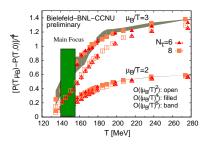




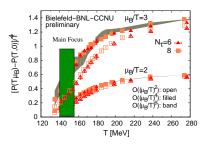
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- Higher order cumulants will also help in bracketing the possible CEP. Current LQCD data suggest it is $\mu_B/T \le 2$ but a larger value cannot be ruled out.